Asset selection with Local Search

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1 Introduction

We provide a code example for a simple asset selection problem. The purpose of this vignette is to provide the code in a convenient way; for more details, please see Gilli et~al. [2011]. We start by attaching the package.

```
> require("NMOF")
> set.seed(112233)
```

2 The problem

We wish to select between K_{inf} and K_{sup} out of n_{A} assets such that an equally-weighted portfolio of these assets has the lowest-possible variance. The formal model is:

$$\min_{w} w'\Sigma w \tag{1}$$

subject to the constraints

$$\begin{aligned} w_j &= {}^1\!/_K \quad \text{for } j \in J\,, \\ K_{\text{inf}} \leq K &\leq K_{\text{sup}}\,. \end{aligned}$$

The weights are stored in the vector w; the symbol J stands for the set of assets in the portfolio; and $K = \#\{J\}$ is the cardinality of this set, ie, the number of assets in the portfolio.

3 Setting up the algorithm

We start by attaching the package and creating random data. We simulate 500~assets: each gets a random volatility between 20% and 40%, and all pairwise correlations are set to~0.6.

```
> na <- 500L
> C <- array(0.6, dim = c(na, na)); diag(C) <- 1
> minVol <- 0.20; maxVol <- 0.40
> Vols <- (maxVol - minVol) * runif(na) + minVol
> Sigma <- outer(Vols, Vols) * C
```

The objective function.

```
> OF <- function(x, data) {
+     xx <- as.logical(x)
+     w <- x/sum(x)
+     res <-  crossprod(w[xx], data$Sigma[xx, xx])
+     res <- tcrossprod(w[xx], res)
+     res
+ }
```

. . . or even simpler:

```
> OF2 <- function(x, data) {
+     xx <- as.logical(x); w <- 1/sum(x)
+     res <- sum(w * w * data$Sigma[xx, xx])
+     res
+ }
```

The neighbourhood function.

```
> neighbour <- function(xc, data) {
+     xn <- xc
+     p <- sample.int(data$na, data$nn, replace = FALSE)
+     xn[p] <- abs(xn[p] - 1L)
+     ## reject infeasible solution
+     if((sum(xn) > data$Ksup) || (sum(xn) < data$Kinf))
+         xc else xn
+ }
```

We collect all necessary information in the list `data`: the variance–corvariance matrix `Sigma`, the cardinality limits˜`Kinf` and `Ksup`, the total number of assets `na` (ie, the cardinality of the asset universe), and the parameter˜`nn`. This parameter controls the neighbourhood: it gives the number of assets that are to be changed when a new solution is computed.

```
> data <- list(Sigma = Sigma,
+              Kinf = 30L,
+              Ksup = 60L,
+                na = na,
+                nn = 1L)
```

4 Solving the model

As an initial solution we use a random portfolio.

```
> card0 <- sample(data$Kinf:data$Ksup, 1L, replace = FALSE)
> assets <- sample.int(na, card0, replace = FALSE)
> x0 <- numeric(na)
> x0[assets] <- 1L
```

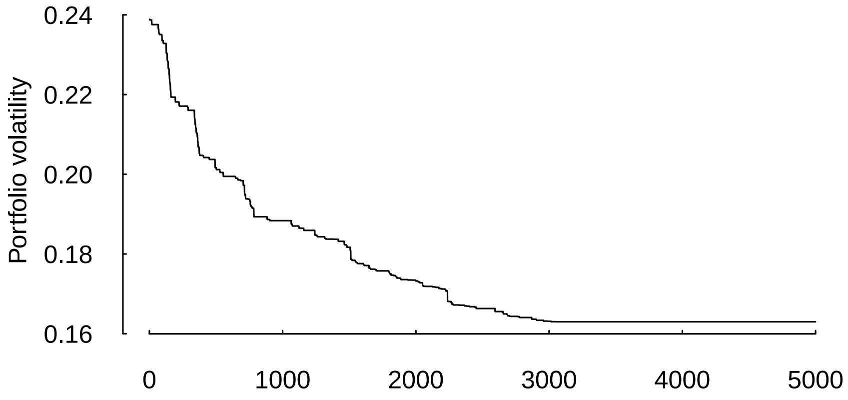
With this implementation we need assume that `data$Ksup > data$Kinf`. (If `data$Ksup == data$Kinf`, then `sample` returns a draw `1:data$Kinf`.)

We collect all settings in the list `algo`.

```
> ## settings
> algo <- list(x0 = x0,
+       neighbour = neighbour,
+              nS = 5000L,
+     printDetail = FALSE,
+        printBar = FALSE)
```

It remains to run the algorithm.

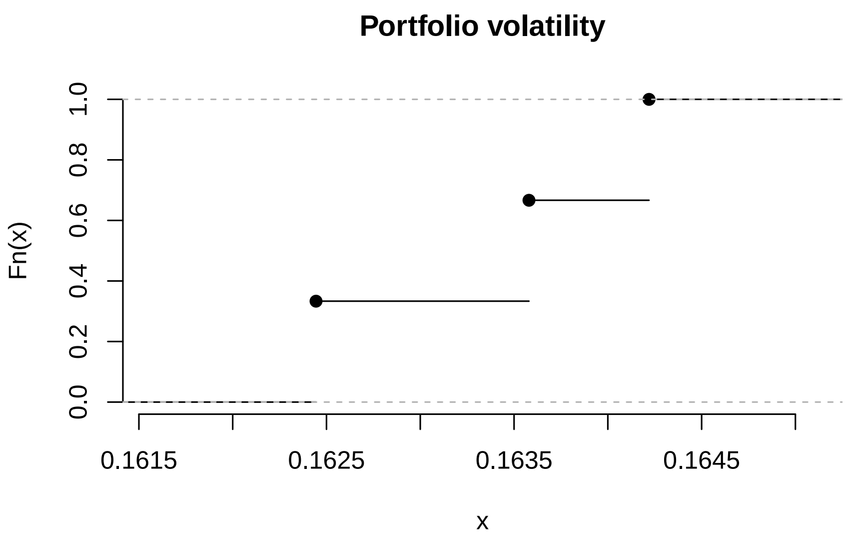
```
> system.time(sol1 <- LSopt(OF, algo, data))

   user  system elapsed
  1.080   0.004   1.082

> sqrt(sol1$OFvalue)

          [,1]
[1,] 0.1629979

> par(ylog = TRUE, bty = "n", las = 1, tck = 0.01)
> plot(sqrt(sol1$Fmat[,2L]),
+      type = "l", xlab = "", ylab = "Portfolio volatility")
```



(Recall that the simulated data had volatilities between 20 and 40%.)

We can also run the search repeatedly with the same starting value.

```
> trials <- 3L
> allRes <- restartOpt(LSopt, n = trials, OF, algo = algo, data = data)
> allResOF <- numeric(trials)
> for (i in seq_len(trials))
+     allResOF[i] <- sqrt(allRes[[i]]$OFvalue)
> par(bty = "n")
> plot(ecdf(allResOF), main = "Portfolio volatility")
```



(We run `LSopt` only three times, to keep the build time for the vignette acceptable. To get more meaningful results, we should run more trials.)

References

Manfred Gilli, Dietmar Maringer, and Enrico Schumann. *Numerical Methods and Optimization in Finance*. Elsevier, 2011.