An Alternative Parameterization for Moult

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This is a brief introduction to the alternative parameterization of the moult model, in which the halfway date, instead of the start of moult, is estimated. We explain the difference in the likelihood and demonstrate differences in the output using the sanderlings data.

moult | feather | primaries

Introduction. In the traditional moult model (Underhill and Zucchini, 1988; Underhill *et al.*, 1990; Erni *et al.*, 2013), the estimates for duration and start of date are highly negatively correlated. This negative correlation indicates difficulties to separate the effects of duration and start of moult. Ideally, parameter estimates should be independent.

In an alternative parameterization of the moult likelihood, instead of mean start of moult, we use a *halfway parameter* μ to define the point in time when 50% of birds have completed 50% of moult. Let h_i denote the time when individual i has completed half of its moult. We again assume that $h_i \sim N(\mu, \sigma^2)$, where σ^2 captures the variability in timing in reaching this halfway point. The duration parameter τ is as before.

This new parameterization with the halfway parameter is more stable and robust to outliers (Les Underhill, pers. comm.).

Likelihood. In terms of the new parameterization, the probability of moult index y_i on day d_i is

$$P(\text{moult score } y_i \text{ on day } d_i) = P\left(h_i = d_i - \tau(y_i - 0.5)\right).$$

We estimate this probability by

$$P\left(h = d - \tau(y - 0.5)\right) = \int_{h=d-\tau(y-prec-0.5)}^{d-\tau(y+prec-0.5)} h dh,$$

where $prec$ is half of the precision with which the moult index has been measured. The probability of being in moult on day d_i is

$$P(\text{in moult on day } d_i | \mu, \sigma^2) = \int_{d_i-0.5\tau}^{d_i+0.5\tau} h_i dh_i$$

i.e. that the halfway date lies within half of the duration of the observed date. Start and end of moult can be derived from these parameters as $\hat{\mu} - 0.5\hat{\tau}$ and $\hat{\mu} + 0.5\hat{\tau}$, respectively.

Example. We use the sanderlings data to compare the correlation matrix of the two parameterizations.

```
library(moult)

m.orig <- moult(MIndex ~ Day, type = 3,
                data = sanderlings)
summary(m.orig)
```

```
#
#  Call:
#  moult(formula = MIndex ~ Day, data = sanderlings, type = 3)
#
#
#  Duration coefficients:
#              Estimate Std. Error
#  intercept.1    99.44      18.53
#
#  Mean start date coefficients:
#            Estimate Std. Error
#  intercept    126.4      8.817
#
#  Coefficients for standard deviation in start date:
#              Estimate Std. Error
#  (Intercept)    17.46      6.187
#
```

```
#  Log-likelihood: -151.6 on 3 Df
```

```
round(cov2cor(vcov(m.orig)), digits = 2)
```

```
#                d.intercept.1 s.intercept sd.(Intercept)
#  d.intercept.1          1.00       -0.85           0.14
#  s.intercept           -0.85        1.00           0.24
#  sd.(Intercept)         0.14        0.24           1.00
```

```
m.alt <- moult_alternative(
  MIndex ~ Day,
  type = 3, data = sanderlings)
summary(m.alt)
```

```
#
#  Call:
#  moult_alternative(formula = MIndex ~ Day, data = sanderlings, type = 3)
#
#
#  Duration coefficients:
#              Estimate Std. Error
#  intercept.1    99.39      18.54
#
#  Mean start date coefficients:
#            Estimate Std. Error
#  intercept    176.1      5.004
#
#  Coefficients for standard deviation in start date:
#              Estimate Std. Error
#  (Intercept)    17.46      6.187
#
#  Log-likelihood: -151.6 on 3 Df
```

```
round(cov2cor(vcov(m.alt)), digits = 2)
```

```
#                d.intercept.1 s.intercept sd.(Intercept)
#  d.intercept.1          1.00        0.36           0.14
#  s.intercept            0.36        1.00           0.69
#  sd.(Intercept)         0.14        0.69           1.00
```

The mean start date parameter estimates start of moult in the original parameterization, and the halfway date in the alternative parameterization. The halfway date should correspond approximately to 'start of moult + 0.5 × duration'. The estimates for duration and standard deviation are very similar. The high negative correlation between duration and start of moult disappears with the alternative parameterization. However, the correlation between the halfway date and the standard deviation estimate has increased. The alternative parameterization is still in testing phase, and we currently do not know much about its behaviour.

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References

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