Proportion: A comprehensive R package for inference on single Binomial proportion and Bayesian computations

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Introduction

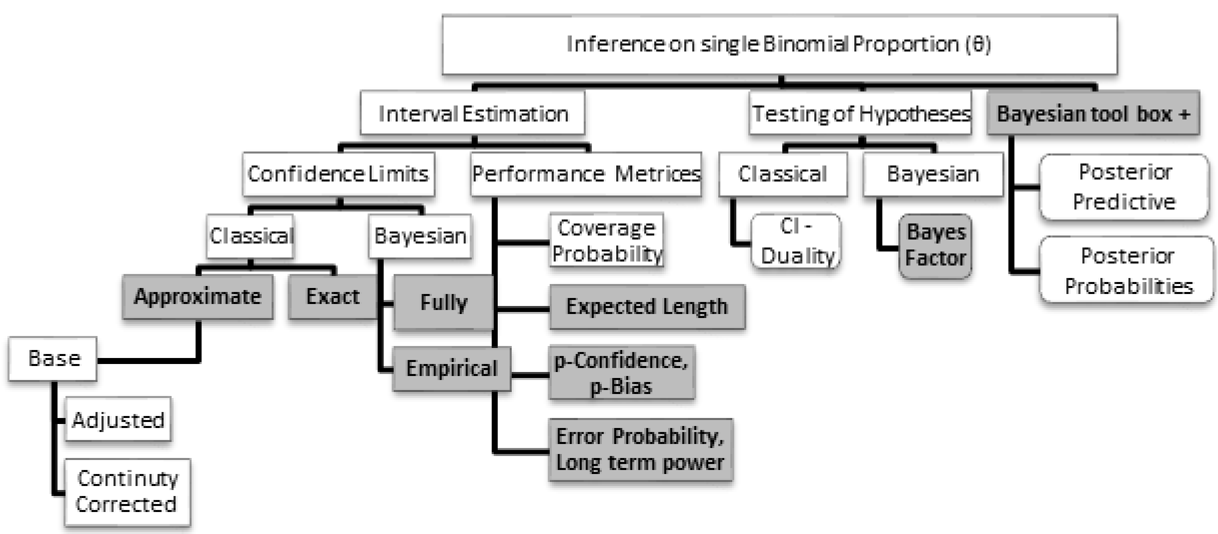
Let x denote the number of successes in n independent Bernoulli trials with X ~ Binomial (n, p) then $\hat{p} = x/n$ denotes the sample proportion. Single binomial proportion (p) has drawn appreciable research attention with theoretical, applied, and pedagogic objectives.

This package has identified scope to collate widely or frequently used methods involved in the inferential problems regarding p and prominent procedures for comparing in terms of their performance. This includes two major statistical paradigms, Classical and Bayesian; especially, later provides a list of tools to broaden this scope.

Objective of this package is to present interval estimation procedures for 'p' outlined above in a more comprehensive way. Performance assessment of these procedures such as coverage probability, Expected length, Error, p-confidence and p-bias are included. Also, an array of Bayesian computations (Bayes factor, Empirical Bayesian, Posterior predictive computation, and posterior probability) with conjugate prior is made available. More importantly package has aimed to complement the summaries using more appropriate graphical forms that enhance the presentation and teaching activities.

Workflow

Following Figure depicts the way this inferential problem can be understood so as to expand the scope of computations; **bold face** indicates modification of existing procedures or addition of new procedures such as t-distribution based Wald method that are not available for wider audience.



Notations Used

1. x: Number of successes
2. n: Number of trials
3. α: Level of significance
4. e: Exact method indicator in [0, 1] {1: Clopper Pearson, 0.5: Mid P}. In all exact functions you can set a range of values between 0 and 1.
5. a and b: Beta parameters for hypothetical parameter generation; Prior parameters in Bayesian predictive models
6. t1 and t2: Limits for tolerance (within which CP lies)
7. π: Population parameter
8. f: Failure limit
9. h: Constant used in adjustment methods
10. c: Constant used in continuity corrected methods
11. a_1, a_2: Prior parameters in Bayesian estimation procedures
12. LL, UL: Lower and Upper limits for the intervals due to any other methods
13. s: Number of simulations
14. hp: Hypothetical parameter values
15. sL, sU: Lower and Upper specification for hyper prior in Empirical Bayesian (EB) approach
16. m, xnew: Number of trials and number of successes in Bayesian predictive models
17. th0, th1, th2: Parameter values in the models M_0, M_1, M_2 of Bayes factor
18. a_j, b_j: Prior parameters in the models $M_j (j = 0, 1, 2)$ of Bayes factor
19. th: Parameter value in Bayesian posterior probabilities

Naming comvention used in the package

Table 1: Naming convention used in functions

Abbrivation	Expansion
ci, ciA, ciC	Confidence Interval, adjusted CI and continuity corrected CI
covp, covpA, covpC	Coverage Probability, adjusted CP & continuity corrected CP
expl, explA, explC	Expected Length, adjusted Expected Length & continuity corrected EL
length, lengthA, lengthC	Sum of Length, adjusted Sum of Length & continuity corrected sumLen
pCOpBI, pCOpBIA, pCOpBIC	p-Confidence & p-Bias, adjusted p-Conf & p-Bias and continuity corrected p-Confidence & p-Bias
err, errA, errC	Error, adjusted error and continuity corrected error
AS	ArcSine
LR	Likelihood Ratio
LT	Logit Wald
SC	Score
TW	Wald-T
WD	Wald
All	6 base methods - Wald, Wald-T, Logit Wald, ArcSine, LR, Score
AAll	6 adj methods - Wald, Wald-T, Logit Wald, ArcSine, LR, Score
CAll	5 cont. corr. methods - Wald, Wald-T, Logit Wald, ArcSine, Score
BA	Bayesian
EX	Exact - setting e=0.5 gives mid-p and e=1 gives Clopper-Pearson

Table 2: Guide to identify core functions - Plot, Modifications and x are optional

Plot	Concept	Modifications	Name	Single x	Sample combination	Sample function
Plot	ci	A	AS	x	ci + A + AS + x =	ciAASx
	covp	C	SC		Plot + ci + A + AS + x =	PlotciAASx
	expl		BA		Plot + covp + C + SC =	PlotcovpCSC
	length		EX		expl + A + TW =	explATW
	pCOpBI		TW		expl + A + TW + x =	explATWx
	err		LT		length + WD =	lenghtWD
			WD		length + A + WD =	lengthAWD
			LR		length + C + WD =	lengthCWD

Confidence Interval

Table 3: Confidence Interval

	Basic	Basic-x	Adj	Adj-x	CC	CC-x
ArcSine	ciAS	ciASx	ciAAS	ciAASx	ciCAS	ciCASx
LR	ciLR	ciLRx	ciALR	ciALRx		
Logit	ciLT	ciLTx	ciALT	ciALTx	ciCLT	ciCLTx
Score	ciSC	ciSCx	ciASC	ciASCx	ciCSC	ciCSCx
Wald-T	ciTW	ciTWx	ciATW	ciATWx	ciCTW	ciCTWx
Wald	ciWD	ciWDx	ciAWD	ciAWDx	ciCWD	ciCWDx
All	ciAll	ciAllx	ciAAll	ciAAllx	ciCAll	ciCAllx
Bayes	ciBA	ciBAx				
Exact	ciEX	ciEXx				

Table 4: Plotting functions of CI

	Basic	Basic-x	Adj	Adj-x	CC	CC-x
ArcSine	PlotciAS		PlotciAAS		PlotciCAS	
LR	PlotciLR		PlotciALR			
Logit	PlotciLT		PlotciALT		PlotciCLT	
Score	PlotciSC		PlotciASC		PlotciCSC	
Wald-T	PlotciTW		PlotciATW		PlotciCTW	
Wald	PlotciWD		PlotciAWD		PlotciCWD	
Allg	PlotciAllg	PlotciAllxg	PlotciAAllg	PlotciAAllxg	PlotciCAllg	PlotciCAllxg
All	PlotciAll	PlotciAllx	PlotciAAll	PlotciAAllx	PlotciCAll	PlotciCAllx
Bayes	PlotciBA					
Exact	PlotciEX	PlotciEXx				

1. CONFIDENCE INTERVAL- BASE METHODS

1. **Wald:**
   Wald-type interval that results from inverting large-sample test and evaluates standard errors at maximum likelihood estimates for all $x = 0, 1, 2..n$.
2. **Score:**
   A score test approach based on inverting the test with standard error evaluated at the null hypothesis

is due to Wilson for all $x = 0, 1, 2..n$.

3. **ArcSine:**
   Wald-type interval for all $x = 0, 1, 2..n$. using the arcsine transformation of the parameter p; that is based on the normal approximation for $sin^{-1}(p)$
4. **Logit Wald:**
   Wald-type interval for all $x = 0, 1, 2..n$. based on the logit transformation of p; that is that is normal approximation for $log\frac{p}{1-p}$
5. **Wald-t:**
   An approximate method based on a t_approximation of the standardized point estimator for all $x = 0, 1, 2..n$.; that is the point estimator divided by its estimated standard error. Essential boundary modification is when $x = 0$ or n, $\hat{p} = \frac{x+2}{n+4}$
6. **Likelihood Ratio:**
   Likelihood ratio limits for all $x = 0, 1, 2..n$. obtained as the solution to the equation in p formed as logarithm of ratio between binomial likelihood at sample proportion and that of over all possible parameters
7. **Exact:**
   Confidence interval for p (for all $x = 0, 1, 2..n$.), based on inverting equal-tailed binomial tests with null hypothesis $H_0 : p = p_0$ and calculated from the cumulative binomial distribution. Exact two sided P-value is usually calculated as $P = 2[ePr(X = x) + minPr(X < x), Pr(X > x)]$ where probabilities are found at null value of p and $0 \leq e \leq 1$.
8. **Bayesian:**
   Highest Probability Density (HPD) and two tailed intervals are provided for all $x = 0, 1, 2..n$ based on the conjugate prior beta (a, b) for the probability of success p of the binomial distribution so that the posterior is beta $(x + a, n - x + b)$.

2.CONFIDENCE INTERVAL- ADJUSTED METHODS

1. **Wald:**
   Given data x and n are modified as $x + h$ and $n + (2 * h)$ respectively, where $h > 0$ then Wald-type interval is applied for all $x = 0, 1, 2..n$.
2. **Score:**
   A score test approach is used after the given data x and n are modified as $x + h$ and $n + (2 * h)$ respectively, where $h > 0$ and for all $x = 0, 1, 2..n$.
3. **ArcSine:**
   Wald-type interval for the arcsine transformation of the parameter p for the modified data $x + h$ and $n + (2 * h)$, where $h > 0$ and for all $x = 0, 1, 2..n$.
4. **Logit Wald:**
   Wald-type interval for the logit transformation $log\frac{p}{1-p}$ of the parameter p for the modified data $x + h$ and $n + (2 * h)$, where $h > 0$ and for all $x = 0, 1, 2..n$.
5. **Wald-t:**
   Given data x and n are modified as $x + h$ and $n + (2 * h)$ respectively, where $h > 0$ then approximate method based on a t_approximation of the standardized point estimator for all $x = 0, 1, 2..n$.
6. **Likelihood Ratio:**
   Likelihood ratio limits for the data $x + h$ and $n + (2 * h)$ instead of the given x and n, where h is a positive integer $(1, 2.)$ and for all $x = 0, 1, 2..n$.

3.CONFIDENCE INTERVAL- CONTINUITY CORRECTED METHODS

1. **Wald:**
   Wald-type interval (for all $x = 0, 1, 2..n$) using the test statistic $\frac{|\hat{p}-p|-c}{SE}$ where $c > 0$ is a constant for continuity correction

2. **Score:**
   A score test approach using the test statistic $\frac{|\hat{p}-p|-c}{SE}$ where $0 < c < 1/(2n)$ is a constant for continuity correction for all $x = 0, 1, 2..n$.
3. **ArcSine:**
   Wald-type interval for the arcsine transformation using the test statistic $\frac{|sin^{-1}\hat{p}-sin^{-1}p|-c}{SE}$ where $c > 0$ is a constant for continuity correction and for all $x = 0, 1, 2..n$.
4. **Logit Wald:**
   Wald-type interval for the logit transformation of the parameter p using the test statistic $\frac{|L(\hat{p})-L(p)|-c}{SE}$ where $c > 0$ is a constant for continuity correction and $L(x) = log\frac{x}{1-x}$ for all $x = 0, 1, 2..n$. Boundary modifications when $x = 0$ or $x = n$ using Exact method values.
5. **Wald-t:**
   Approximate method based on a t_approximation of the standardized point estimator using the test statistic $\frac{|\hat{p}-p|-c}{SE}$ where $c > 0$ is a constant for continuity correction for all $x = 0, 1, 2..n$. Boundary modifications when $x = 0$ or $x = n$ using Wald adjustment method with $h = 2$.

Coverage Probability

Table 5: Coverage Probability

	Basic	Adjusted	Continuity corrected
ArcSine	covpAS	covpAAS	covpCAS
LR	covpLR	covpALR	
Logit	covpLT	covpALT	covpCLT
Score	covpSC	covpASC	covpCSC
Wald-T	covpTW	covpATW	covpCTW
Wald	covpWD	covpAWD	covpCWD
All	covpAll	covpAAll	covpCAll
Bayes	covpBA		
Exact	covpEX		

Table 6: Plotting functions of Coverage Probability

	Basic	Adjusted	Continuity corrected
ArcSine	PlotcovpAS	PlotcovpAAS	PlotcovpCAS
LR	PlotcovpLR	PlotcovpALR	
Logit	PlotcovpLT	PlotcovpALT	PlotcovpCLT
Score	PlotcovpSC	PlotcovpASC	PlotcovpCSC
Wald-T	PlotcovpTW	PlotcovpATW	PlotcovpCTW
Wald	PlotcovpWD	PlotcovpAWD	PlotcovpCWD
All	PlotcovpAll	PlotcovpAAll	PlotcovpCAll
Bayes	PlotcovpBA		
Exact	PlotcovpEX		

4.Metric 1:COVERAGE PROBABILITY (Applicable to Base, Adjusted and Continuity Corrected Methods)

1. **Wald:**
   Evaluation of Wald-type interval using coverage probability, root mean square statistic, and the proportion of proportion lies within the desired level of coverage

2. **Score:**
   Evaluation of score test approach using coverage probability, root mean square statistic, and the proportion of proportion lies within the desired level of coverage
3. **ArcSine:**
   Evaluation of Wald-type interval for the arcsine transformation of the parameter p using coverage probability, root mean square statistic, and the proportion of proportion lies within the desired level of coverage
4. **Logit Wald:**
   Evaluation of Wald-type interval based on the logit transformation of p using coverage probability, root mean square statistic, and the proportion of proportion lies within the desired level of coverage
5. **Wald-t:**
   Evaluation of approximate method based on a t_approximation of the standardized point estimator using coverage probability, root mean square statistic, and the proportion of proportion lies within the desired level of coverage
6. **Likelihood Ratio:**
   Evaluation of Likelihood ratio limits using coverage probability, root mean square statistic, and the proportion of proportion lies within the desired level of coverage
7. **Exact:**
   Evaluation of Confidence interval for p based on inverting equal-tailed binomial tests with null hypothesis H0: p = p0 using coverage probability, root mean square statistic, and the proportion of proportion lies within the desired level of coverage.
8. **Bayesian:**
   Evaluation of Bayesian Highest Probability Density (HPD) and two tailed intervals using coverage probability, root mean square statistic, and the proportion of proportion lies within the desired level of coverage for the Beta - Binomial conjugate prior model for the probability of success p.

Length

Table 7: Sum of length

	SumLen	Adj-SumLen	CC-SumLen
ArcSine	lengthAS	lengthAAS	lengthCAS
LR	lengthLR	lengthALR	
Logit	lengthLT	lengthALT	lengthCLT
Score	lengthSC	lengthASC	lengthCSC
Wald-T	lengthTW	lengthATW	lengthCTW
Wald	lengthWD	lengthAWD	lengthCWD
All	lengthAll	lengthAAll	lengthCAll
Bayes	lengthBA		
Exact	lengthEX		

Table 8: Plotting functions of sum length and expected length (EL)

	SumLen	EL	Adj-SumLen	Adj-EL	CC-SumLen	CC-EL
ArcSine	PlotlengthAS	PlotexplAS	PlotlengthAAS	PlotexplAAS	PlotlengthCAS	PlotexplCAS
LR	PlotlengthLR	PlotexplLR	PlotlengthALR	PlotexplALR		
Logit	PlotlengthLT	PlotexplLT	PlotlengthALT	PlotexplALT	PlotlengthCLT	PlotexplCLT
Score	PlotlengthSC	PlotexplSC	PlotlengthASC	PlotexplASC	PlotlengthCSC	PlotexplCSC
Wald-T	PlotlengthTW	PlotexplTW	PlotlengthATW	PlotexplATW	PlotlengthCTW	PlotexplCTW
Wald	PlotlengthWD	PlotexplWD	PlotlengthAWD	PlotexplAWD	PlotlengthCWD	PlotexplCWD
All	PlotlengthAll	PlotexplAll	PlotlengthAAll	PlotexplAAll	PlotlengthCAll	PlotexplCAll

	SumLen	EL	Adj-SumLen	Adj-EL	CC-SumLen	CC-EL
Bayes	PlotlengthBA	PlotexplBA				
Exact	PlotlengthEX	PlotexplEX				

4.Metric 2:EXPECTED LENGTH (Applicable to Base, Adjusted and Continuity Corrected Methods)

1. **Wald:**
   Evaluation of Wald-type intervals using expected length of the $n+1$ intervals
2. **Score:**
   Evaluation of score test approach using expected length of the $n+1$ intervals
3. **ArcSine:**
   Evaluation of Wald-type interval for the arcsine transformation of the parameter p using expected length of the $n+1$ intervals
4. **Logit Wald:**
   Evaluation of Wald-type interval based on the logit transformation of p using expected length of the $n+1$ intervals
5. **Wald-t:**
   Evaluation of approximate method based on a t_approximation of the standardized point estimator using expected length of the $n+1$ intervals
6. **Likelihood Ratio:**
   Evaluation of Likelihood ratio limits using expected length of the $n+1$ intervals
7. **Exact:**
   Evaluation of Confidence interval for p based on inverting equal-tailed binomial tests with null hypothesis $H_0 : p = p_0$ using expected length of the $n+1$ intervals.
8. **Bayesian:**
   Evaluation of Bayesian Highest Probability Density (HPD) and two tailed intervals using expected length of the $n+1$ intervals for the Beta - Binomial conjugate prior model for the probability of success p.

p-Confidence & p-Bias

Table 9: p-Confidence & p-Bias

	Basic	Adjusted	Continuity corrected
ArcSine	pCOpBIAS	pCOpBIAAS	pCOpBICAS
LR	pCOpBILR	pCOpBIALR	
Logit	pCOpBILT	pCOpBIALT	pCOpBICLT
Score	pCOpBISC	pCOpBIASC	pCOpBICSC
Wald-T	pCOpBITW	pCOpBIATW	pCOpBICTW
Wald	pCOpBIWD	pCOpBIAWD	pCOpBICWD
All	pCOpBIAll	pCOpBIAAll	pCOpBICAll
Bayes	pCOpBIBA		
Exact	pCOpBIEX		

Table 10: Plotting functions for p-Confidence & p-Bias

	Basic	Adjusted	Continuity corrected
ArcSine	PlotpCOpBIAS	PlotpCOpBIAAS	PlotpCOpBICAS

	Basic	Adjusted	Continuity corrected
LR	PlotpCOpBILR	PlotpCOpBIALR	
Logit	PlotpCOpBILT	PlotpCOpBIALT	PlotpCOpBICLT
Score	PlotpCOpBISC	PlotpCOpBIASC	PlotpCOpBICSC
Wald-T	PlotpCOpBITW	PlotpCOpBIATW	PlotpCOpBICTW
Wald	PlotpCOpBIWD	PlotpCOpBIAWD	PlotpCOpBICWD
All	PlotpCOpBIAll	PlotpCOpBIAAll	PlotpCOpBICAll
Bayes	PlotpCOpBIBA		
Exact	PlotpCOpBIEX		

5.Metric 3:p-CONFIDENCE, p-BIAS (BASE METHOD)

1. **Wald:**
   Evaluation of Wald-type intervals using p-confidence and p-bias for the $n+1$ intervals
2. **Score:**
   Evaluation of score test approach using p-confidence and p-bias for the $n+1$ intervals
3. **ArcSine:**
   Evaluation of Wald-type interval for the arcsine transformation of the parameter p using p-confidence and p-bias for the $n+1$ intervals
4. **Logit Wald:**
   Evaluation of Wald-type interval based on the logit transformation of p using p-confidence and p-bias for the $n+1$ intervals
5. **Wald-t:**
   Evaluation of approximate method based on a t_approximation of the standardized point estimator using p-confidence and p-bias for the $n+1$ intervals
6. **Likelihood Ratio:**
   Evaluation of Likelihood ratio limits using p-confidence and p-bias for the $n+1$ intervals
7. **Exact:**
   Evaluation of Confidence interval for p based on inverting equal-tailed binomial tests with null hypothesis $H_0 : p = p_0$ using p-confidence and p-bias for the $n+1$ intervals.
8. **Bayesian:**
   Evaluation of Bayesian Highest Probability Density (HPD) and two tailed intervals using p-confidence and p-bias for the $n+1$ intervals for the Beta - Binomial conjugate prior model for the probability of success p.

6.Metric 3:p-CONFIDENCE, p-BIAS (Applicable to Base, Adjusted and Continuity Corrected Methods)

1. **Wald:**
   Evaluation of adjusted Wald-type interval using p-confidence and p-bias for the $n+1$ intervals
2. **Score:**
   Evaluation of adjusted score test approach using p-confidence and p-bias for the $n+1$ intervals
3. **ArcSine:**
   Evaluation of adjusted Wald-type interval for the arcsine transformation of the parameter p using p-confidence and p-bias for the $n+1$ intervals
4. **Logit Wald:**
   Evaluation of adjusted Wald-type interval based on the logit transformation of p using p-confidence and p-bias for the $n+1$ intervals
5. **Wald-t:**
   Evaluation of approximate and adjusted method based on a t_approximation of the standardized point estimator using p-confidence and p-bias for the $n+1$ intervals

6. **Likelihood Ratio:**
   Evaluation of adjusted Likelihood ratio limits using p-confidence and p-bias for the $n+1$ intervals

Error and long term power

Table 11: Error and long term power

	Basic	Adjusted	Continuity corrected
ArcSine	errAS	errAAS	errCAS
LR	errLR	errALR	
Logit	errLT	errALT	errCLT
Score	errSC	errASC	errCSC
Wald-T	errTW	errATW	errCTW
Wald	errWD	errAWD	errCWD
All	errAll	errAAll	errCAll
Bayes	errBA		
Exact	errEX		

Table 12: Plotting functions for error and long term power

	Basic	Adjusted	Continuity corrected
ArcSine	PloterrAS	PloterrAAS	PloterrCAS
LR	PloterrLR	PloterrALR	
Logit	PloterrLT	PloterrALT	PloterrCLT
Score	PloterrSC	PloterrASC	PloterrCSC
Wald-T	PloterrTW	PloterrATW	PloterrCTW
Wald	PloterrWD	PloterrAWD	PloterrCWD
All	PloterrAll	PloterrAAll	PloterrCAll
Bayes	PloterrBA		
Exact	PloterrEX		

7.Metric 4:ERROR (Applicable to Base, Adjusted and Continuity Corrected Methods)

1. **Wald:**
   Evaluation of Wald-type intervals using error due to the difference of achieved and nominal level of significance for the $n+1$ intervals
2. **Score:**
   Evaluation of score test approach using error due to the difference of achieved and nominal level of significance for the $n+1$ intervals
3. **ArcSine:**
   Evaluation of Wald-type interval for the arcsine transformation of the parameter p error due to the difference of achieved and nominal level of significance for the $n+1$ intervals
4. **Logit Wald:**
   Evaluation of Wald-type interval based on the logit transformation of p using error due to the difference of achieved and nominal level of significance for the $n+1$ intervals
5. **Wald-t:**
   Evaluation of approximate method based on a t_approximation of the standardized point estimator using error due to the difference of achieved and nominal level of significance for the $n+1$ intervals
6. **Likelihood Ratio:**
   Evaluation of Likelihood ratio limits using error due to the difference of achieved and nominal level of

significance for the $n+1$ intervals

7. **Exact:**
   Evaluation of Confidence interval for p based on inverting equal-tailed binomial tests with null hypothesis $H_0 : p = p_0$ using error due to the difference of achieved and nominal level of significance for the $n+1$ intervals.
8. **Bayesian:**
   Evaluation of Bayesian Highest Probability Density (HPD) and two tailed intervals using error due to the difference of achieved and nominal level of significance for the $n+1$ intervals for the Beta - Binomial conjugate prior model for the probability of success p.

8. EVALUATION METHODS FOR GENERAL APPROACH

1. Evaluation of intervals obtained from any method using coverage probability, root mean square statistic, and the proportion of proportion lies within the desired level of coverage for the $n+1$ intervals and pre-defined space for the parameter p using Monte Carle simulation
2. Graphical evaluation of intervals obtained from any method using coverage probability, root mean square statistic, and the proportion of proportion lies within the desired level of coverage for the $n+1$ intervals and pre-defined space for the parameter p using Monte Carle simulation

Additional functions

Table 13: Additional functions

Hypothesis	covp	length	pCOpBI	Others	Error
hypotestBAF1	covpGEN	lengthGEN	pCOpBIGEN	empericalBA	errGEN
hypotestBAF1x	PlotcovpGEN	PlotlengthGEN	PlotpCOpBIGEN	empericalBAx	
hypotestBAF2x	covpSIM	lengthSIM		probPOSx	
hypotestBAF2	PlotcovpSIM	PlotlengthSIM		probPOS	
hypotestBAF3x		PlotexplGEN		probPREx	
hypotestBAF3		PlotexplSIM		probPRE	
hypotestBAF4x					
hypotestBAF4					
hypotestBAF5x					
hypotestBAF5					
hypotestBAF6x					
hypotestBAF6					

9. Testing of hypothesis and other functions for Bayesian method

1. **EBA:**
   Highest Probability Density (HPD) and two tailed intervals are provided for all $x = 0, 1, 2..n$ based on empirical Bayesian approach for Beta-Binomial model. Lower and Upper support values are needed to obtain the MLE of marginal likelihood for prior parameters.
2. **probPRE:**
   Computes posterior predictive probabilities for the required size of number of trials (m) from the given number of trials (n) for the given parameters for Beta prior distribution
3. **hypotestBAF1:**
   Computes Bayes factor under Beta-Binomial model for the model: $H_0 : p = p_0$ Vs $H_A : p \neq p_0$ from the given number of trials n and for all number of successes $x = 0, 1, 2......n$
4. **hypotestBAF2:**
   Computes Bayes factor under Beta-Binomial model for the model: $H_0 : p = p_0$ Vs $H_A : p > p_0$ from

the given number of trials n and for all number of successes $x = 0, 1, 2......n$

5. **hypotestBAF3:**
   Computes Bayes factor under Beta-Binomial model for the model: $H_0 : p = p_0$ Vs $H_A : p < p_0$ from the given number of trials n and for all number of successes $x = 0, 1, 2......n$
6. **hypotestBAF4:**
   Computes Bayes factor under Beta-Binomial model for the model: $H_0 : p \leq p_0$ Vs $H_A : p > p_0$ from the given number of trials n and for all number of successes $x = 0, 1, 2......n$
7. **hypotestBAF5:**
   Computes Bayes factor under Beta-Binomial model for the model: $H_0 : p \geq p_0$ Vs $H_A : p < p_0$ from the given number of trials n and for all number of successes $x = 0, 1, 2......n$
8. **hypotestBAF6:**
   Computes Bayes factor under Beta-Binomial model for the model: $H_0 : p < p_1$ Vs $H_A : p > p_2$ from the given number of trials n and for all number of successes $x = 0, 1, 2......n$
9. **probPOS:**
   Computes probability of the event $p < p_0$ (p_0 is specified in $[0, 1]$) based on posterior distribution of Beta-Binomial model with given parameters for prior Beta distribution for all $x = 0, 1, 2...n$ (n: number of trials)

10. Assistance for reading papers

We have taken six key papers and shown how this package can assist in reproducing the results in these papers. On top of that we have also provided some further areas researchers can gain insight using the package.

Table 14: Additional functions

#	x	n	Paper	Methods	Function	Additional options
1	20	0	Newcombe	Wald ,Score,(both with	ciWDx,ciSCx	Methods such as Bayesian,Arcsine,
2	29	1		and without CC) Exact	ciCWDx,	Logit Wald methods; Numerical
3	148	15		and LR for CI	ciCSCx,	& graphical comparisons of methods
4	263	81			ciEXx,ciLRx	Use of general CC and adj. factor
5	10	10	Joseph 2005	Wald and Exact CI	ciWDx,ciEXx	Bayes factor
6	98	100				
7	17	16	Zhou 2008	Wald, Score,	ciWDx, ciSCx	Other methods such as Bayesian,
8	14	12		Agresti-Coull &	ciAWDx	Arcsine Logit transformed methods
				modified logit for CI		Use of general CC and adj. factor
9	167	0	Wei 2012	Score, Agresti-Coull	ciSCx,	Other classical methods; Numerical
				Bayesian(Jeffreys prior)	ciAWDx,	& graphical comparisons of methods
				& other two methods	ciBAx	Use of general CC and adj. factor
10	109	16	Tuyl 2008	Bayesian method with	ciBAx	Other classical methods; Numerical
				five different beta priors		& graphical comparisons of methods
						Use of general CC and adj. factor
11	NA	10	Vos 2005	p-confidence, p-bias	pCOpBIBA	

Paper 1 (Newcombe 1998):

The paper has compared seven methods (Wald, Wald continuity corrected, Likelihood ratio, Score (Wilson), Score, continuity corrected, Clopper Pearson, Mid-P) for Two-sided confidence intervals for the single proportion. Evaluation criteria, Average CP, Aberrations, Zero Width Interval and Non Coverage aspects are considered. Four illustrative data sets have also been provided The package, proportion provides a more comprehensive way of summarizing results similar to the above studies; for example, a function ($ciAllxforn = 20, x = 0$) from the package yields an easily comparable summary (numerical and graphical) together with other useful measures like existence of aberration, zero width intervals (ZWI). ArcSine and Wald-t methods are additional inclusions; Summaries / Methods which are not readily available elsewhere such as opting with Exact method in a more general way (ciEXx), continuity corrected (ciCAllx), or adding pseudo constants (ciAAllx) in a more general way or Quantile (Q) based and Highest Posterior (H) based CI from Bayesian conjugate method (ciBAx) with an option for specifying any plausible value for the two parameters of prior beta distribution.

Numerical Summaries

Table 15: Asymptotic methods CI using ciAllx(x=0,n=20,alp=0.05)

method	x	LowerLimit	UpperLimit	LowerAbb	UpperAbb	ZWI
Wald	0	0.0000000	0.0000000	NO	NO	YES
ArcSine	0	0.0472546	0.0472546	NO	NO	YES
Likelihood	0	0.0000253	0.0916153	NO	NO	NO
Score	0	0.0000000	0.1611252	NO	NO	NO
Logit-Wald	0	0.0000000	0.1684335	NO	NO	NO
Wald-T	0	0.0000000	0.2440055	YES	NO	NO

Table 16: Exact method CI using ciBAx(x=0,n=20,alp=0.05,e=c(0.1,0.5,0.95,1))

x	LEXx	UEXx	LABB	UABB	ZWI	e
0	0	0.0669670	NO	NO	NO	0.10
0	0	0.1391083	NO	NO	NO	0.50
0	0	0.1662980	NO	NO	NO	0.95
0	0	0.1684335	NO	NO	NO	1.00

Table 17: Bayesian CI using ciBAx() with x=0,n=20,alp=0.05, varying a(2,1,0.05,0.02 and b(2,1,0.05,2)

Desc	x	LBAQx	UBAQx	LBAHx	UBAHx
Assuming Symmetry	0	0.0107100	0.2194866	0.0023218	0.1913698
Flat	0	0.0012049	0.1610976	0.0000000	0.1329459
Jeffreys	0	0.0000242	0.1166390	0.0000000	0.0904764
Near boundary	0	0.0000000	0.0089203	0.0000000	0.0021319

Table 18: Adding Pseudo constant using ciAAllx(x=0,n=20,alp=0.05,h=2)

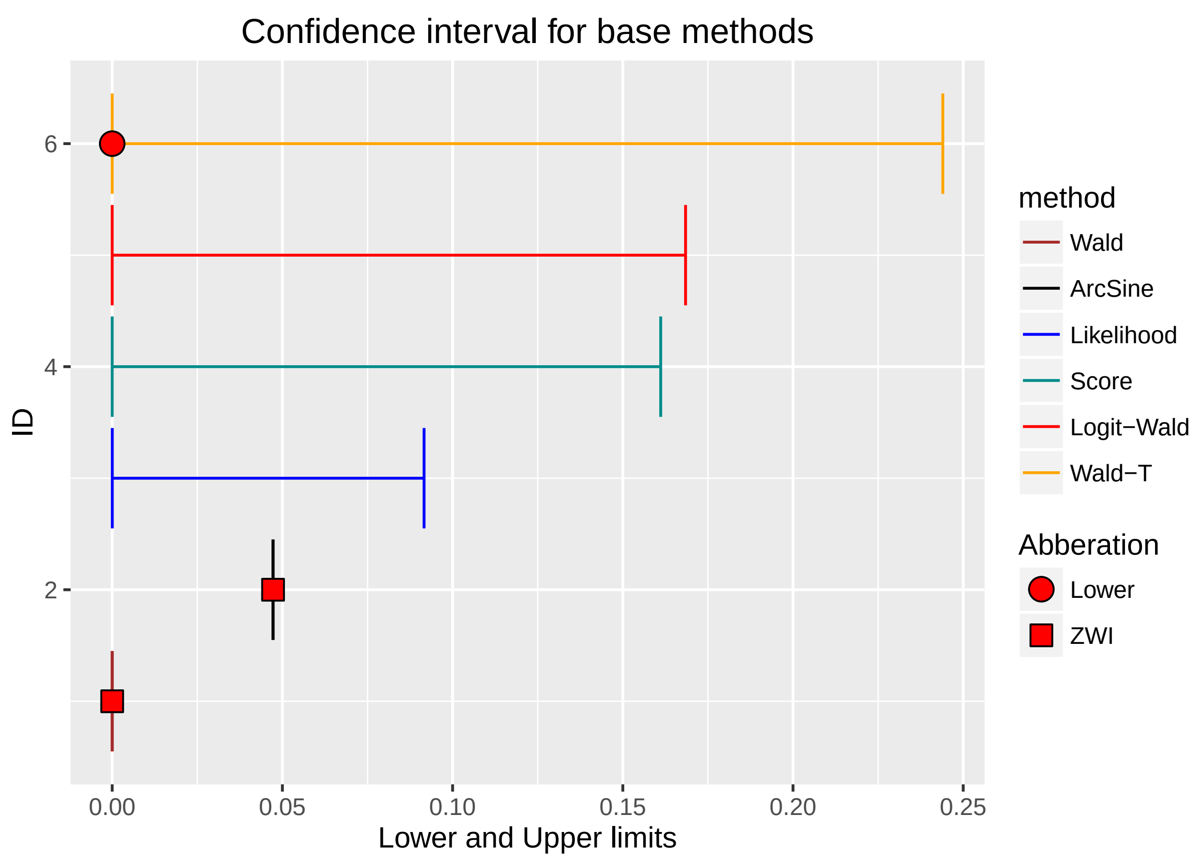
method	x	LowerLimit	UpperLimit	LowerAbb	UpperAbb	ZWI
Adj-Wald	0	0.0000000	0.1939085	YES	NO	NO
Adj-ArcSine	0	0.0085880	0.2238858	NO	NO	NO
Adj-Liklihood	0	0.0143776	0.2357444	NO	NO	NO
Adj-Score	0	0.0231588	0.2584880	NO	NO	NO
Adj-Logit Wald	0	0.0209299	0.2788112	NO	NO	NO
Adj-Wald-T	0	0.0000000	0.2231950	YES	NO	NO

Table 19: Adding Continuity Correction, c = 1/(2n) & using ciCAllx(x=0,n=20,alp=0.05,c=1/40)

method	x	LowerLimit	UpperLimit	LowerAbb	UpperAbb	ZWI
Wald	0	0.0000000	0.0250000	YES	NO	NO
ArcSine	0	0.0584251	0.0584251	NO	NO	YES
Score	0	0.0045747	0.2004533	NO	NO	NO
Logit Wald	0	0.0000000	0.1684335	NO	NO	NO
Wald-T	0	0.0000000	0.2690055	YES	NO	NO

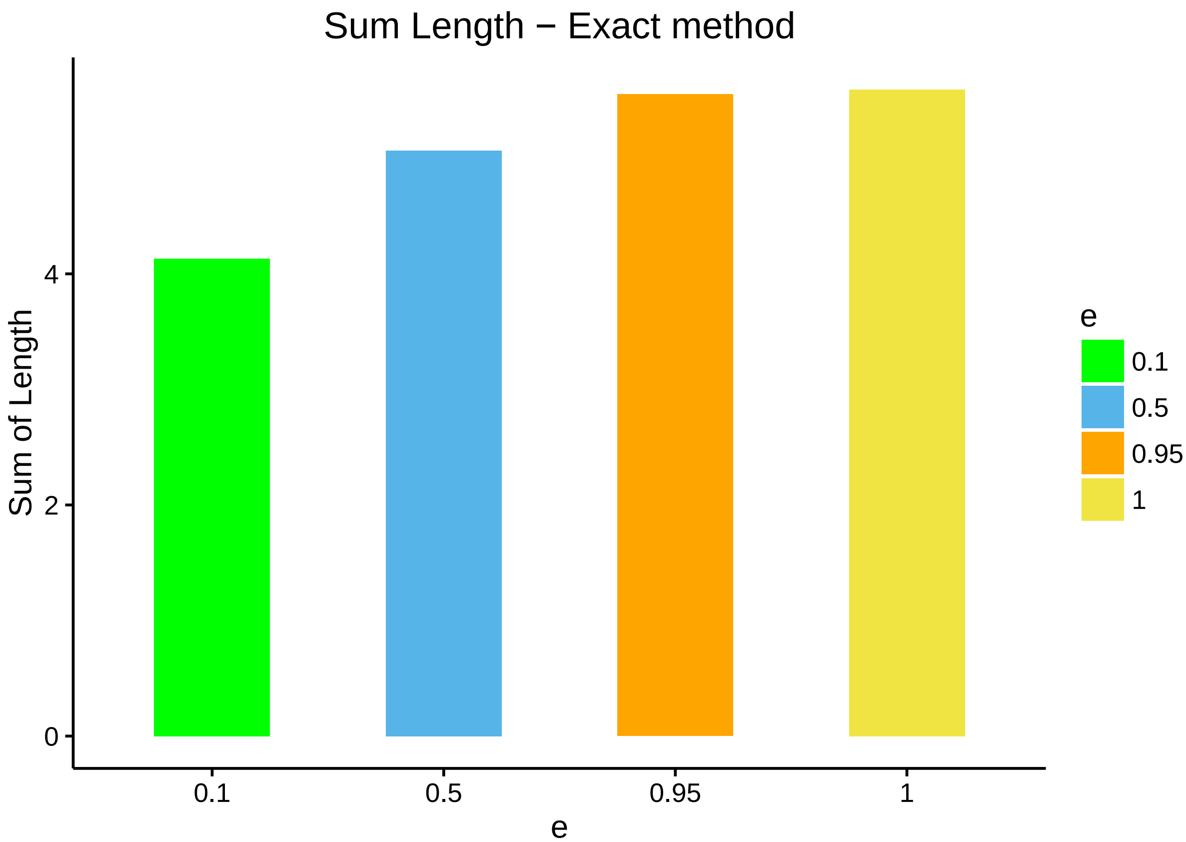
Graphical Summaries

```
# 17. Paper 1
PlotciAllx(x=0,n=20,alp=0.05)
```



Corresponding comparison for sum of length of CI can be obtained as below

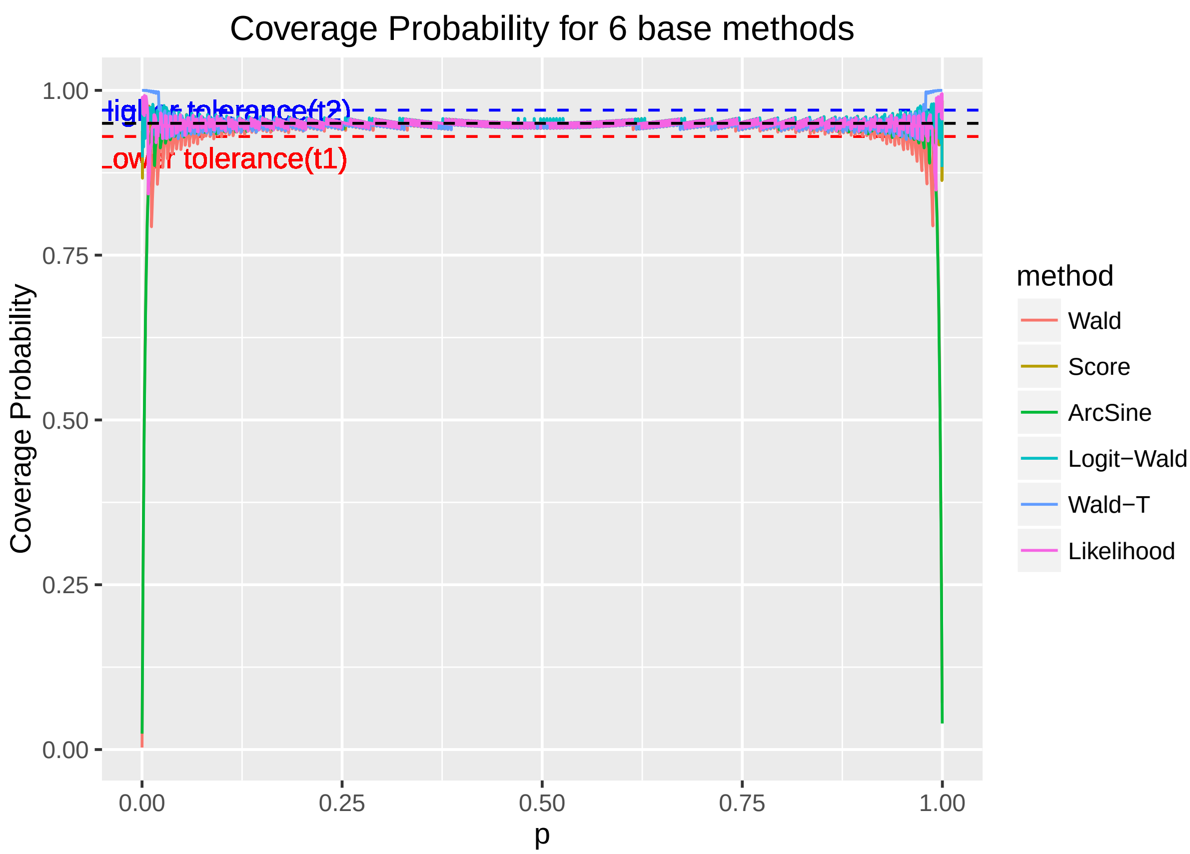
```
# 18. Plot of sum of length of exact method
PlotlengthEX(n=10,alp=0.05,e=c(0.1,0.5,0.95,1),a=1,b=1)
```



In the case of other evaluation criteria, package proportion provides ample scope for comparing competing methods. Following table and plots illustrate for n = 250 (inspired from n = 263) using the functions $covpAll(n, alp, a1, b1)$ and $PlotcovpAll(n, alp, a1, b1)$

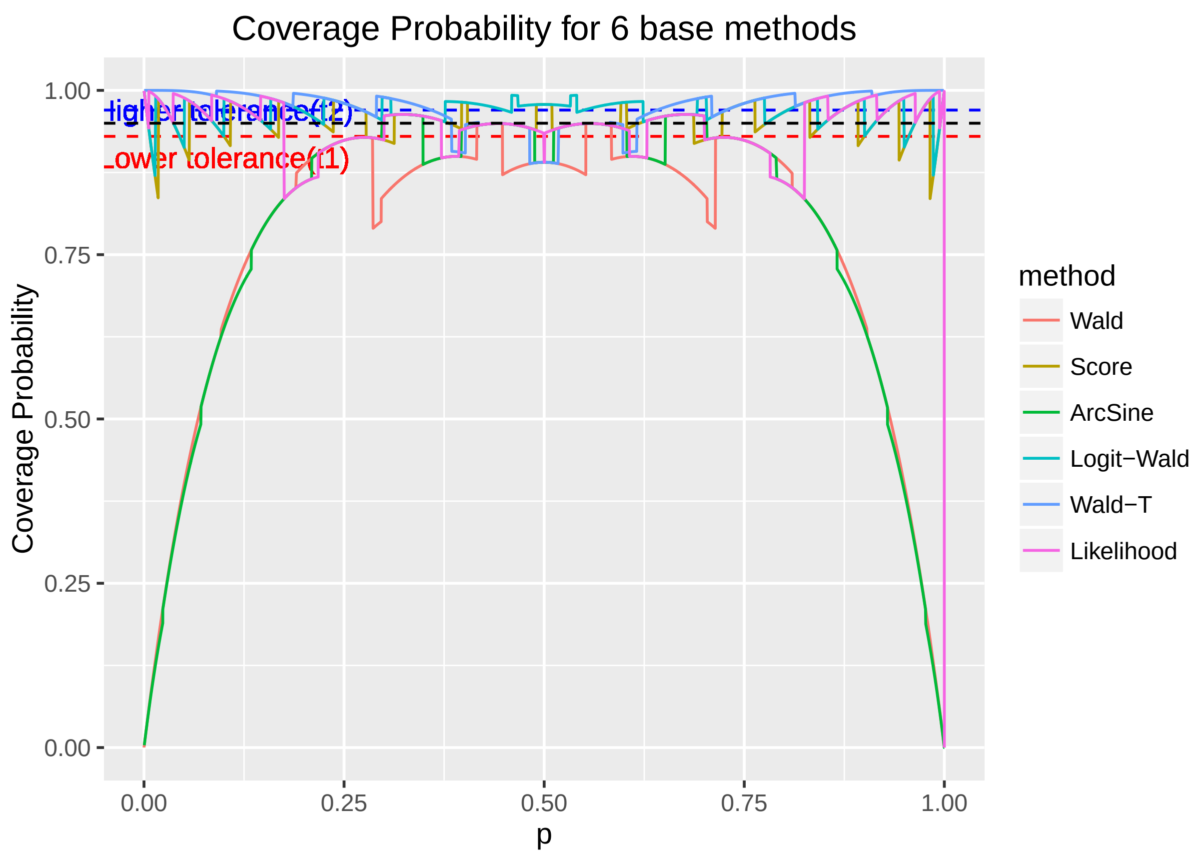
Table 20: Coverage probability using covpAll()

method	MeanCP	MinCP	RMSE_N	RMSE_M	RMSE_MI	tol
Wald	0.9378760	0.0865404	0.0549425	0.0535881	0.8530206	91.20
ArcSine	0.9393519	0.0270436	0.0731636	0.0723846	0.9151754	95.62
Lilelihood	0.9492731	0.8448036	0.0086118	0.0085811	0.1048213	97.20
Score	0.9505127	0.8466798	0.0056706	0.0056474	0.1039863	98.98
WaldLogit	0.9521169	0.8797497	0.0063260	0.0059613	0.0726123	98.08
Wald-T	0.9500755	0.9238705	0.0115417	0.0115414	0.0286340	94.76



For a more comparative case consider similar plot for $n = 10$

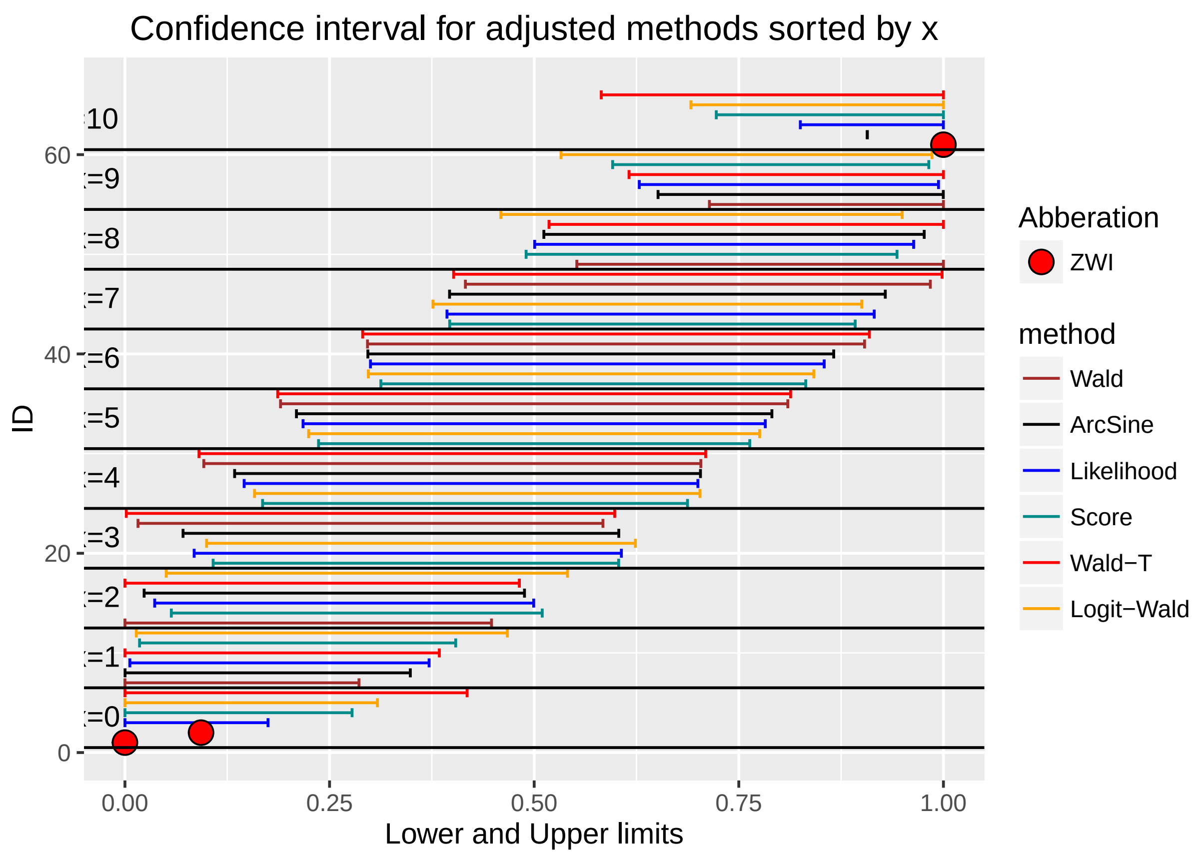
```
# 20. Paper 1
PlotcovpAll(n=10,alp=0.05,a=1,b=1,t1=0.93,t2=0.97)
```



Paper 2 (Joseph and Reinfold 2005):

A tutorial kind of article pertaining to obtain CI based on inverting two tailed tests involving single proportion is available in Joseph and Reinfold 2005. This mainly deals with Wald large sample and Exact methods for CI and hypothesis testing involving values near boundary of p. Only interval for Wald is avialble in the paper however, comparison of procedure would enhance the presentation and purpose. One way is through a pictorial output can be improved further by sorting the CI for each $x = 0, 1, .., n$ using a function $PlotciAllg(n, alp)$.

```
# 21. Paper 2 - display the function
PlotciAllg(n=10,alp=0.05)
```



Further, one of the significant features of the package is readily available Bayesian testing alternatives involving single binomial proportion (p). For example data from this paper involves a classical testing $H_0 : p \leq 0.9$ vs. $H_1 : p > 0.9$, Bayes factor can be calculated using the function $hypotestBAF4(x, n, th0, a0, b0, a1, b1)$. (Six functions are available for the exhaustive possibilities of testing hypotheses on p). Numerical result for this data under the assumption that uniform and Jeffreys prior for null and alternate models respectively is 0.0832, which is evident to reject H_0.

Additionally this package has an option (like hypotestBAF4) to compare Bayes factor for all possible values of x (such as the one listed below) so as to understand the possible change in the values of Bayes factor in turn the decision.

```
# 20. Paper 2 - display the function
hypotestBAF4(n=10, th0=0.9, a0=1,b0=1,a1=0.5,b1=0.5)
```

Table 21: Hypothesis test, H0: p <= 0.9 vs. H1: p > 0.9

x	BaFa01	Interpretation
0	3.987416e+10	Evidence against H1 is very strong
1	2.088687e+08	Evidence against H1 is very strong
2	3.619818e+06	Evidence against H1 is very strong
3	1.165063e+05	Evidence against H1 is very strong
4	5.924426e+03	Evidence against H1 is very strong
5	4.440244e+02	Evidence against H1 is very strong
6	4.749817e+01	Evidence against H1 is strong
7	7.123142e+00	Evidence against H1 is positive
8	1.449043e+00	Evidence against H1 is not worth more than a bare mention

x	BaFa01	Interpretation
9	3.619376e-01	Evidence against H0 is not worth more than a bare mention
10	8.319770e-02	Evidence against H0 is positive

Paper 3 (Zhou et al 2008) and 4 (Wei Yu et al 2012):

The main objective of Zhou et al 2008 is to improve logit Wald method and the method has been illustrated with $x = 16$ and $n = 17$. Similarly, Wei Yu et al 2012 have attempted an improvement for Score method with a real data example ($x = 16, n = 109$). Further, two adjustment methods can easily be compared with other adjustment methods using the options available from the package ciAAllx. Intentionally the adjustment factor (h) is taken as zero to compare with original results of respective studies. Such comparison is pervasive in a statistical investigation involving a parameter, particularly for p.

```
# 20. Function to evaluate ci varying the adding constant h
ciAAllx(x=16, n=17,alp = 0.05,h=0)
ciAAllx(x=16, n=109,alp = 0.05,h=0)
```

The full results are shown below with h values of 0,1 and 2.

Table 22: CI with x=16, n=17 & h=0

method	x	LowerLimit	UpperLimit	LowerAbb	UpperAbb	ZWI
Adj-Wald	16	0.8293268	1.0000000	NO	YES	NO
Adj-ArcSine	16	0.7845775	0.9999467	NO	NO	NO
Adj-Liklihood	16	0.7656344	0.9965448	NO	NO	NO
Adj-Score	16	0.7301797	0.9895396	NO	NO	NO
Adj-Logit Wald	16	0.6796805	0.9917795	NO	NO	NO
Adj-Wald-T	16	0.7478459	1.0000000	NO	YES	NO

Table 23: CI with x=16, n=109 & h=0

method	x	LowerLimit	UpperLimit	LowerAbb	UpperAbb	ZWI
Adj-Wald	16	0.0803520	0.2132260	NO	NO	NO
Adj-ArcSine	16	0.0869474	0.2190422	NO	NO	NO
Adj-Liklihood	16	0.0888655	0.2211349	NO	NO	NO
Adj-Score	16	0.0924191	0.2252076	NO	NO	NO
Adj-Logit Wald	16	0.0919145	0.2262616	NO	NO	NO
Adj-Wald-T	16	0.0788708	0.2147072	NO	NO	NO

Table 24: CI with x=16, n=17 & h=1

method	x	LowerLimit	UpperLimit	LowerAbb	UpperAbb	ZWI
Adj-Wald	16	0.7567438	1.0000000	NO	YES	NO
Adj-ArcSine	16	0.7221104	0.9888902	NO	NO	NO
Adj-Liklihood	16	0.7090774	0.9816599	NO	NO	NO
Adj-Score	16	0.6860592	0.9706414	NO	NO	NO
Adj-Logit Wald	16	0.6626006	0.9735380	NO	NO	NO
Adj-Wald-T	16	0.7243814	1.0000000	NO	YES	NO

Table 25: CI with x=16, n=109 & h=1

method	x	LowerLimit	UpperLimit	LowerAbb	UpperAbb	ZWI
Adj-Wald	16	0.0861567	0.2201496	NO	NO	NO
Adj-ArcSine	16	0.0925269	0.2257484	NO	NO	NO
Adj-Liklihood	16	0.0943876	0.2277104	NO	NO	NO
Adj-Score	16	0.0978748	0.2316357	NO	NO	NO
Adj-Logit Wald	16	0.0973834	0.2326298	NO	NO	NO
Adj-Wald-T	16	0.0847926	0.2215137	NO	NO	NO

Table 26: CI with x=16, n=17 & h=2

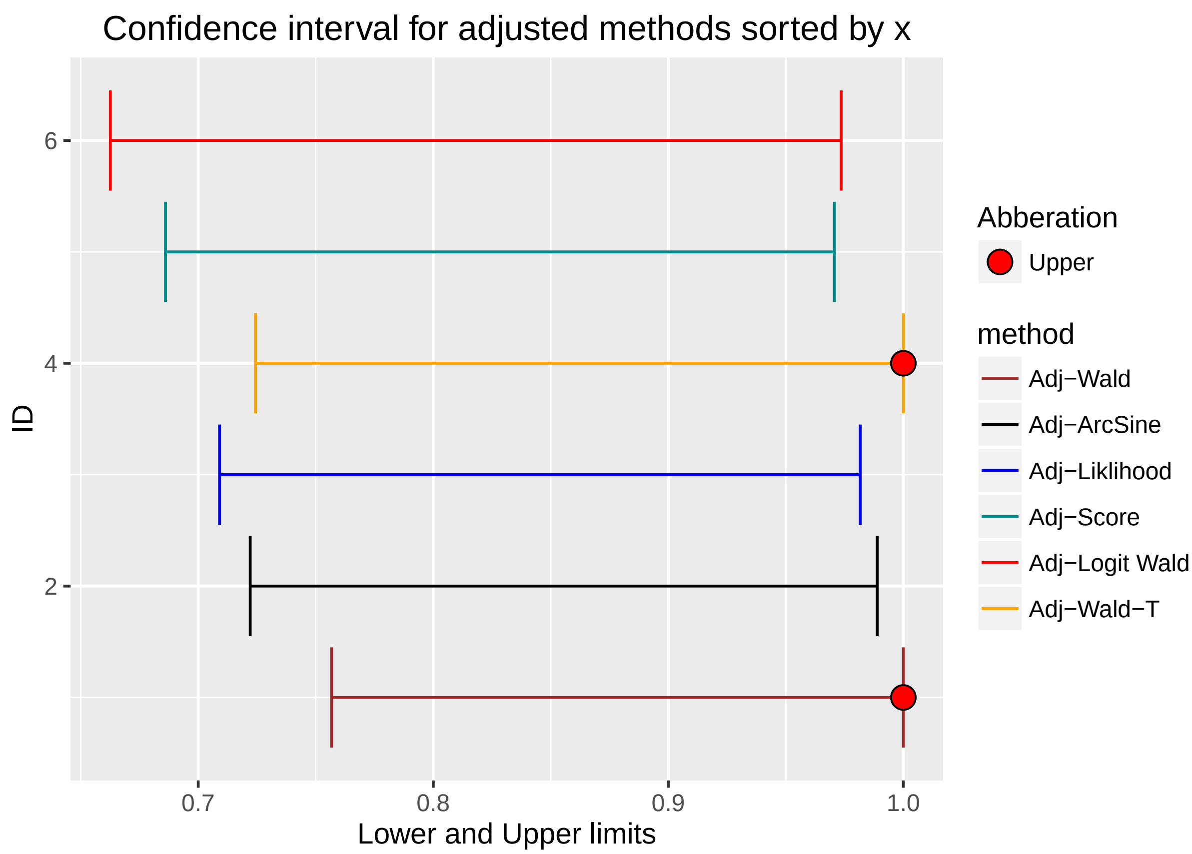
method	x	LowerLimit	UpperLimit	LowerAbb	UpperAbb	ZWI
Adj-Wald	16	0.7074793	1.0000000	NO	YES	NO
Adj-ArcSine	16	0.6798301	0.9701145	NO	NO	NO
Adj-Liklihood	16	0.6701048	0.9624273	NO	NO	NO
Adj-Score	16	0.6536394	0.9501899	NO	NO	NO
Adj-Logit Wald	16	0.6386495	0.9532031	NO	NO	NO
Adj-Wald-T	16	0.6887962	1.0000000	NO	YES	NO

Table 27: CI with x=16, n=109 & h=2

method	x	LowerLimit	UpperLimit	LowerAbb	UpperAbb	ZWI
Adj-Wald	16	0.0918193	0.2267648	NO	NO	NO
Adj-ArcSine	16	0.0979757	0.2321580	NO	NO	NO
Adj-Liklihood	16	0.0998091	0.2340433	NO	NO	NO
Adj-Score	16	0.1032005	0.2377869	NO	NO	NO
Adj-Logit Wald	16	0.1027218	0.2387274	NO	NO	NO
Adj-Wald-T	16	0.0905595	0.2280246	NO	NO	NO

To compare the length of the intervals for the data $x = 16, n = 17$, a graphical form can be obtained from the package using

```
# 21. Paper 3&4 - Plot of all the adjusted CI with h=1
PlotciAAllxg(x=16,n=17,alp=0.05,h=1)
```



As can be seen above the grouping function (ending with g) convinently orders the results within each value of x.

Another aspect is the way Exact method has been handled; based on the extensive studies for adjusting Exact method, this package confines to randomized test using the constant e in [0, 1]. Example 6 (Joseph and Reinfold 2005- see table above) may be reproduced with the function ciEXx as shown below.

```
# 22. Paper 3&4 - display the function
ciEXx(x=98, n=100,alp = 0.05,e=c(0.1,.5,0.95,1))
```

Table 28: CI-Exact with x=98, n=100

x	LEXx	UEXx	LABB	UABB	ZWI	e
98	0.9429629	0.9946812	NO	NO	NO	0.10
98	0.9355021	0.9966313	NO	NO	NO	0.50
98	0.9300850	0.9975033	NO	NO	NO	0.95
98	0.9295962	0.9975662	NO	NO	NO	1.00

Paper 5 (Tuyl et al 2008):

This paper has compared difference non-informative priors with an informative prior based on an earlier study for single binomial proportion with a real data set $x = 0, n = 167$. This is one of most often cited examples for zero successes which have witnessed active research. The predictive density based comparison has been carried out to emphasize a specific prior assumption. This package provides readily available options in Bayesian computation using posterior predictive distributions for a wider comparison and probabilities. A quick comparison using Uniform prior for zero successes or possibility for $p = 0.5$ can be explored using the function $probPREx(x, n, xnew, m, a1, a2)$. The variable xnew and m varies, keeping x=0, n=167, a1=a2=1.

Table 29: Predicted probability with x=0, n=167 varying xnew and m

x	n	xnew	m	preprb
0	167	0	10	0.9438202
0	167	0	50	0.7706422
0	167	0	100	0.6268657
0	167	0	150	0.5283019
0	167	5	10	0.0000002
0	167	25	50	0.0000000
0	167	50	100	0.0000000
0	167	75	150	0.0000000

Assuming that the example depicts a rare event, an analysis with posterior probabilities would enhance the analysis when the function $probPOSx(x, n, a, b, th)$ is used

Table 30: Guidance for priors used below

Description	a	b
Uniform prior	1.000	1.00
Jeffreys prior	0.500	0.50
Tuyl p1	0.042	27.96
Tuyl p2	1.000	666.00
Tuyl p3	1.000	398.00

Table 31: Posterior probability with x=0, n=167 varying th

	Uniform Prior	Jeffreys prior	Tuyl p1	Tuyl p2	Tuyl p3
th=0.001	0.1547172	0.4370766	0.9479747	0.5654381	0.4318005
th=0.01	0.8151954	0.9332767	0.9976693	0.9997687	0.9965811
th=0.1	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
th=0.5	0.9998190	0.9999656	0.9999998	1.0000000	1.0000000

Also literature often compiles the frequentist evaluation criteria for Bayesian methods too and hence this package includes most prominent methods as well as other measures as a sign of enlarging the scope of comparison.

Paper 6 (Vos and Hudson 2005):

The p-confidence and p-Bias from Vos and Hudson (2005) and the result for p-confidence and p-bias for two types of Bayesian CI for $n = 10$ using $pCOpBIBA(n, alp, a1, a2)$ is

Table 32: p-Confidence & p-Bias of Bayesian method for n=10, a1=a2=1

x1	pconfQ	pbiasQ	pconfH	pbiasH
1	58.75446	33.416269	86.04498	1.7034458
2	76.33893	15.007198	86.58091	1.4986566
3	82.59405	7.832379	87.14676	1.0040862
4	85.83952	3.477491	87.57388	0.4968235
5	87.87691	0.000000	87.87691	0.0000000
6	85.83952	3.477491	87.57388	0.4968235
7	82.59405	7.832379	87.14676	1.0040859
8	76.33893	15.007198	86.58091	1.4986566
9	58.75446	33.416269	86.04498	1.7034458

References

[1] 1998 Newcombe RG. Two-sided confidence intervals for the single proportion: Comparison of seven methods. Statistics in Medicine: 17; 857 - 872.

[2] 2005 Joseph L and Reinhold C. Statistical Inference for Proportions American Journal of Radiologists 184; 1057 - 1064

[3] 2008 Zhou, X. H., Li, C.M. and Yang, Z. Improving interval estimation of binomial proportions. Phil. Trans. R. Soc. A, 366, 2405-2418

[4] 2012 Wei Yu, Xu Guo and Wangli Xua. An improved score interval with a modified midpoint for a binomial proportion, Journal of Statistical Computation and Simulation, 84, 5, 1-17