

Package ‘NPBBBdesigns’

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Title Construction and a-Efficiency of Nested Partially Balanced Bipartite Block Designs

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Description Construction and evaluation of nested partially balanced bipartite block (NPBBB) designs for comparing a set of test treatments with a set of control treatments under a nested (blocks within blocks) structure. Six systematic construction methods are provided: composing partially balanced bipartite block designs with nested balanced incomplete block designs; augmenting nested partially balanced incomplete block designs with controls; merging rows of group-divisible nested designs; direct construction from group-divisible schemes; and expansion of partially balanced incomplete block designs (Vinayaka et al. 2026: In press). The A-efficiencies of the block and sub-block classifications are computed against the A-optimal completely symmetric reference design, following the test-versus-control optimality framework of Gupta and Parsad (1996) <doi:10.1080/03610929608831743> and Vinayaka et al. (2024) <doi:10.1080/03610926.2024.2024.2024>. The designs are particularly suited to agricultural, animal husbandry, industrial, and clinical trials involving multiple standard checks under nested experimental conditions, such as multi-environment trials where field heterogeneity (blocks) and within-field variation (sub-blocks) must be controlled simultaneously.

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a_value	<i>A-Value of an NPBBB Design for Test-Versus-Control Contrasts</i>
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Description

Returns the trace of the variance-covariance matrix of the estimators of the $v_1 v_2$ elementary test-versus-control contrasts $\tau_i - \tau_j$ (i a test treatment, j a control treatment), that is, the sum of their variances. When the information matrix is completely symmetric within the test set and within the control set (which holds for the A-optimal members of the catalogue) the value is computed in closed form from the canonical quantities f_1, f_2, f_4, f_5 (the average diagonal and off-diagonal entries of the test-test and control-control sub-matrices of C). This reproduces the values reported in the design catalogues of Vinayaka et al. (2026); see also Hedayat and Majumdar (1984) and Stufken (1988).

Usage

```
a_value(design, v1, v2)
```

Arguments

design	A matrix (or data frame) whose rows are the blocks or sub-blocks and whose entries are the treatment labels. Test treatments must be labelled 1, ..., v1 and control treatments v1 + 1, ..., v1 + v2.
v1	Number of test treatments.
v2	Number of control treatments.

Value

A single numeric value, the A-value (sum of variances of the $v_1 v_2$ test-versus-control elementary contrasts).

References

Hedayat AS, Majumdar D (1984) A-optimal incomplete block designs for test treatment-control comparisons. *Technometrics*, 26, 363–370.

Stufken J (1988) On bounds for the efficiency of block designs for comparing test treatments with a control. *Journal of Statistical Planning and Inference*, 19, 361–372.

Vinayaka, Parsad R, Mandal BN, LN Vinaykumar (2026) Nested partially balanced bipartite block designs for comparing test treatments with multiple controls. *Journal of Statistical Theory and Practice*. (In press).

Examples

```
d <- rbind(c(1, 2, 5, 6), c(3, 4, 5, 6))
a_value(d, v1 = 4, v2 = 2)
```

a_value_optimal

A-Value of the A-Optimal Completely Symmetric Reference Design

Description

Computes the smallest attainable A-value (sum of variances of the $v_1 v_2$ test-versus-control elementary contrasts) within the class of connected (sub-)block designs that are completely symmetric in the test and in the control treatments, for a control replication r_0 . This is the benchmark against which the A-efficiency is measured. The expression is the nested-design analogue of the Hedayat-Majumdar / Stufken optimal A-value; see Vinayaka et al. (2026).

Usage

```
a_value_optimal(v1, v2, b, k, r0)
```

Arguments

v1	Number of test treatments.
v2	Number of control treatments.
b	Number of blocks (or sub-blocks) in the classification.
k	Block (or sub-block) size.
r0	Replication of each control treatment in the classification.

Value

A single numeric value, the optimal (minimum) A-value.

References

Hedayat AS, Majumdar D (1984) A-optimal incomplete block designs for test treatment-control comparisons. *Technometrics*, 26, 363–370.

Stufken J (1988) On bounds for the efficiency of block designs for comparing test treatments with a control. *Journal of Statistical Planning and Inference*, 19, 361–372.

Vinayaka, Parsad R, Mandal BN, LN Vinaykumar (2026) Nested partially balanced bipartite block designs for comparing test treatments with multiple controls. *Journal of Statistical Theory and Practice*. (In press).

Examples

```
# Optimal block-classification A-value for an NPBBB with v1 = 9, v2 = 2
a_value_optimal(v1 = 9, v2 = 2, b = 6, k = 10, r0 = 12)
```

construct_method1	<i>Construct an NPBBB Design by Composing a PBBB Design with an NBIB Design</i>
-------------------	---

Description

Implements Method 1 of Vinayaka et al. (2026). Each block of a partially balanced bipartite block (PBBB) design of size k' is replaced by a copy of a nested balanced incomplete block (NBIB) design on $v^* = k'$ symbols, by identifying the k' symbols of the NBIB design with the k' entries of the PBBB block.

Usage

```
construct_method1(pbbb_blocks, nbib_subblocks, q, v2 = 2)
```

Arguments

pbbb_blocks	A list of integer vectors of common length k' , the blocks of a PBBB design, with test treatments labelled $1, \dots, v1$ and controls $v1 + 1, \dots, v1 + v2$.
nbib_subblocks	A list of integer vectors over the symbols $1, \dots, k'$ giving the <i>sub-blocks</i> of an NBIB design, supplied in block order (q sub-blocks per block).
q	Number of sub-blocks per block of the NBIB design.
v2	Number of control treatments in the PBBB design (default 2).

Value

An object of class "npbbb": a list with the following components:

- **method**: a character string naming the construction used.
- **v1, v2**: numbers of test and control treatments.
- **parameters**: a list of the design parameters $v1, v2, b1, b2, r1, r2, k1, k2, q$.

- **block_design**: an integer matrix with b_1 rows and k_1 columns; each row is a block written as its concatenated sub-blocks.
- **subblock_design**: an integer matrix with b_2 rows and k_2 columns; each row is a sub-block.
- **E1, E2**: block and sub-block A-efficiencies.
- **efficiency**: an object of class "npbbb_efficiency" holding the underlying A-values and optimal references.

References

Vinayaka, Parsad R, Mandal BN, LN Vinaykumar (2026) Nested partially balanced bipartite block designs for comparing test treatments with multiple controls. *Journal of Statistical Theory and Practice*. (In press).

Examples

```
# PBBB design: 4 test treatments (1-4), 2 controls (5, 6); block size k' = 4
pbbb <- list(c(1,2,5,6), c(1,3,5,6), c(1,4,5,6),
            c(2,3,5,6), c(2,4,5,6), c(3,4,5,6))
# NBIB on k' = 4 symbols from a one-factorisation of K4 (q = 2 sub-blocks/block)
nbib <- list(c(1,2), c(3,4), c(1,3), c(2,4), c(1,4), c(2,3))
d <- construct_method1(pbbb, nbib, q = 2, v2 = 2)
d
```

construct_method2	<i>Construct an NPBBB Design by Augmenting an NPBIB Design with Controls</i>
-------------------	--

Description

Implements Method 2 of Vinayaka et al. (2026). Starting from a nested partially balanced incomplete block (NPBIB) design with q sub-blocks per block, v_2 control treatments are added to every sub-block. Because the controls appear in every sub-block, every test treatment meets every control the same number of times and every pair of controls co-occurs the same number of times; the resulting design is completely symmetric in the controls and, whenever the parent NPBIB design is itself well balanced, A-optimal for both classifications.

Usage

```
construct_method2(npbib_subblocks, q, v2 = 2)
```

Arguments

npbib_subblocks	A list of integer vectors, one per sub-block of the parent NPBIB design, giving its test treatments labelled $1, \dots, v_1$. Sub-blocks must be supplied in block order: the first q entries form block 1, the next q entries block 2, and so on.
q	Number of sub-blocks per block of the parent NPBIB design.
v2	Number of control treatments to add (default 2).

Value

An object of class "npbbb": a list with the following components:

- **method**: a character string naming the construction used.
- **v1, v2**: numbers of test and control treatments.
- **parameters**: a list of the design parameters v_1 , v_2 , b_1 , b_2 , r_1 , r_2 , k_1 , k_2 , q .
- **block_design**: an integer matrix with b_1 rows and k_1 columns; each row is a block written as its concatenated sub-blocks.
- **subblock_design**: an integer matrix with b_2 rows and k_2 columns; each row is a sub-block.
- **E1, E2**: block and sub-block A-efficiencies.
- **efficiency**: an object of class "npbbb_efficiency" holding the underlying A-values and optimal references.

References

Vinayaka, Parsad R, Mandal BN, LN Vinaykumar (2026) Nested partially balanced bipartite block designs for comparing test treatments with multiple controls. *Journal of Statistical Theory and Practice*. (In press).

Examples

```
# L2-type NPBBB on v = 9 (Example 3.2 of Vinayaka et al. (2026))
sub <- list(c(1,2,3), c(4,5,6), c(1,2,3), c(7,8,9), c(4,5,6), c(7,8,9),
           c(1,4,7), c(2,5,8), c(1,4,7), c(3,6,9), c(2,5,8), c(3,6,9))
d <- construct_method2(sub, q = 2, v2 = 2)
d
```

construct_method3	<i>Construct an NPBBB Design by Merging Rows of a Group-Divisible NPBBB Design</i>
-------------------	--

Description

Implements Method 3 of Vinayaka et al. (2026). In a group-divisible NPBBB design on $v = mn$ treatments arranged in an $m \times n$ array, the treatments of v_2 selected rows are each merged into a single control treatment. The test treatments are the $n(m - v_2)$ remaining array entries, relabelled $1, \dots, v_1$; the merged rows become controls $v_1 + 1, \dots, v_1 + v_2$. A control may occur more than once in a sub-block (see Note 3.1 of the paper); such multiplicities are retained.

Usage

```
construct_method3(
  gd_npbib_subblocks,
  m,
  n,
  q,
```

```

merge_rows = seq_len(v2),
v2 = 2
)

```

Arguments

gd_npbib_subblocks	A list of integer vectors, one per sub-block of the parent group-divisible NPBIB design, with treatments labelled 1, . . . , mn. Sub-blocks must be supplied in block order.
m	Number of rows of the group-divisible scheme.
n	Number of treatments per row.
q	Number of sub-blocks per block.
merge_rows	Integer vector of length v2 giving the rows to merge into controls (default: the first v2 rows).
v2	Number of control treatments (default 2).

Value

An object of class "npbbb": a list with the following components:

- **method**: a character string naming the construction used.
- **v1, v2**: numbers of test and control treatments.
- **parameters**: a list of the design parameters v1, v2, b1, b2, r1, r2, k1, k2, q.
- **block_design**: an integer matrix with b1 rows and k1 columns; each row is a block written as its concatenated sub-blocks.
- **subblock_design**: an integer matrix with b2 rows and k2 columns; each row is a sub-block.
- **E1, E2**: block and sub-block A-efficiencies.
- **efficiency**: an object of class "npbbb_efficiency" holding the underlying A-values and optimal references.

References

Vinayaka, Parsad R, Mandal BN, LN Vinaykumar (2026) Nested partially balanced bipartite block designs for comparing test treatments with multiple controls. *Journal of Statistical Theory and Practice*. (In press).

Examples

```

# Group-divisible NPBIB on v = m*n = 8 treatments (m = 4 rows, n = 2),
# with q = 2 sub-blocks per block; merge the first two rows into v2 = 2 controls
gd <- list(c(1,3), c(2,4), c(5,7), c(6,8), c(1,5), c(2,6),
          c(3,7), c(4,8), c(1,7), c(2,8), c(3,5), c(4,6))
d <- construct_method3(gd, m = 4, n = 2, q = 2, merge_rows = c(1, 2), v2 = 2)
d

```

construct_method4 *Construct an NPBBB Design Directly from a Group-Divisible Scheme*

Description

Implements Method 4 of Vinayaka et al. (2026). The $v_1 = mn$ test treatments are arranged in an $m \times n$ array. For each row, n sub-blocks are formed, each consisting of one treatment from that row together with all v_2 controls; the n sub-blocks of a row constitute a block. The construction yields an A-optimal design for both classifications.

Usage

```
construct_method4(m, n, v2 = 2)
```

Arguments

<code>m</code>	Number of rows (groups) of the group-divisible scheme.
<code>n</code>	Number of treatments per row.
<code>v2</code>	Number of control treatments (default 2).

Value

An object of class "npbbb": a list with the following components:

- **method**: a character string naming the construction used.
- **v1, v2**: numbers of test and control treatments.
- **parameters**: a list of the design parameters $v_1, v_2, b_1, b_2, r_1, r_2, k_1, k_2, q$.
- **block_design**: an integer matrix with b_1 rows and k_1 columns; each row is a block written as its concatenated sub-blocks.
- **subblock_design**: an integer matrix with b_2 rows and k_2 columns; each row is a sub-block.
- **E1, E2**: block and sub-block A-efficiencies.
- **efficiency**: an object of class "npbbb_efficiency" holding the underlying A-values and optimal references.

References

Vinayaka, Parsad R, Mandal BN, LN Vinaykumar (2026) Nested partially balanced bipartite block designs for comparing test treatments with multiple controls. *Journal of Statistical Theory and Practice*. (In press).

Examples

```
d <- construct_method4(m = 4, n = 3, v2 = 2)
d
```

construct_method5	<i>Construct an NPBBB Design by Expanding the Size-4 Blocks of a PBIB Design</i>
-------------------	--

Description

Implements Method 5 of Vinayaka et al. (2026) (specific to $v_2 = 2$). Each block (x_1, x_2, x_3, x_4) of a PBIB design with block size 4 is expanded, with the two controls $0_1, 0_2$, into four blocks of size 6 (each consisting of two sub-blocks of size 3): the i -th block places x_i with both controls in one sub-block and the remaining three treatments in the other.

Usage

```
construct_method5(pbib_blocks)
```

Arguments

`pbib_blocks` A list of integer vectors of length 4, the blocks of a PBIB design with test treatments labelled $1, \dots, v_1$.

Value

An object of class "npbbb" with $v_2 = 2$: a list with the following components:

- **method**: a character string naming the construction used.
- **v1, v2**: numbers of test and control treatments.
- **parameters**: a list of the design parameters $v_1, v_2, b_1, b_2, r_1, r_2, k_1, k_2, q$.
- **block_design**: an integer matrix with b_1 rows and k_1 columns; each row is a block written as its concatenated sub-blocks.
- **subblock_design**: an integer matrix with b_2 rows and k_2 columns; each row is a sub-block.
- **E1, E2**: block and sub-block A-efficiencies.
- **efficiency**: an object of class "npbbb_efficiency" holding the underlying A-values and optimal references.

References

Vinayaka, Parsad R, Mandal BN, LN Vinaykumar (2026) Nested partially balanced bipartite block designs for comparing test treatments with multiple controls. *Journal of Statistical Theory and Practice*. (In press).

Examples

```
# Singular GD design S1: v = 6, b = 3, k = 4
s1 <- list(c(1,2,3,4), c(1,2,5,6), c(3,4,5,6))
d <- construct_method5(s1)
d
```

construct_method6	<i>Construct an NPBBB Design by Expanding the Size-2 Blocks of a PBIB Design</i>
-------------------	--

Description

Implements Method 6 of Vinayaka et al. (2026) (specific to $v_2 = 2$). Each block (x_1, x_2) of a PBIB design with block size 2 is expanded, with the two controls $0_1, 0_2$, into three blocks of size 4 (two sub-blocks of size 2): $[(x_1, x_2); (0_1, 0_2)]$, $[(x_1, 0_1); (x_2, 0_2)]$ and $[(x_1, 0_2); (x_2, 0_1)]$.

Usage

```
construct_method6(pbib_blocks)
```

Arguments

`pbib_blocks` A list of integer vectors of length 2, the blocks of a PBIB design with test treatments labelled $1, \dots, v_1$.

Value

An object of class "npbbb" with $v_2 = 2$: a list with the following components:

- **method**: a character string naming the construction used.
- **v1, v2**: numbers of test and control treatments.
- **parameters**: a list of the design parameters $v_1, v_2, b_1, b_2, r_1, r_2, k_1, k_2, q$.
- **block_design**: an integer matrix with b_1 rows and k_1 columns; each row is a block written as its concatenated sub-blocks.
- **subblock_design**: an integer matrix with b_2 rows and k_2 columns; each row is a sub-block.
- **E1, E2**: block and sub-block A-efficiencies.
- **efficiency**: an object of class "npbbb_efficiency" holding the underlying A-values and optimal references.

References

Vinayaka, Parsad R, Mandal BN, LN Vinaykumar (2026) Nested partially balanced bipartite block designs for comparing test treatments with multiple controls. *Journal of Statistical Theory and Practice*. (In press).

Examples

```
# Semi-regular GD design SR1: v = 4, b = 4, k = 2
sr1 <- list(c(1,3), c(1,4), c(2,3), c(2,4))
d <- construct_method6(sr1)
d
```

info_matrix	<i>Information Matrix of a (Sub-)Block Design for Test-Versus-Control Comparisons</i>
-------------	---

Description

Computes the reduced (treatment) information matrix $C = R - NK^{-1}N'$ for a single classification (blocks or sub-blocks) of a nested partially balanced bipartite block (NPBBB) design. Here R is the diagonal matrix of treatment replications, N is the treatment-by-(sub-)block incidence matrix and K is the diagonal matrix of (sub-)block sizes. Control treatments may occur more than once in a (sub-)block (for example, designs obtained by merging rows of a group-divisible scheme); such multiplicities are counted in N so that C is the correct information matrix under the homoscedastic fixed-effects nested model. For more details see Vinayaka et al. (2026).

Usage

```
info_matrix(design, v1, v2)
```

Arguments

design	A matrix (or data frame) whose rows are the blocks or sub-blocks and whose entries are the treatment labels. Test treatments must be labelled 1, ..., v1 and control treatments v1 + 1, ..., v1 + v2.
v1	Number of test treatments.
v2	Number of control treatments.

Value

A numeric $(v1 + v2)$ by $(v1 + v2)$ information matrix C .

References

Vinayaka, Parsad R, Mandal BN, LN Vinaykumar (2026) Nested partially balanced bipartite block designs for comparing test treatments with multiple controls. *Journal of Statistical Theory and Practice*. (In press).

Examples

```
# Two blocks of size 4 on 4 test treatments (1-4) and 2 controls (5, 6)
d <- rbind(c(1, 2, 5, 6), c(3, 4, 5, 6))
info_matrix(d, v1 = 4, v2 = 2)
```

npbbb_efficiency *A-Efficiency of an NPBBB Design*

Description

Evaluates the A-efficiency of a nested partially balanced bipartite block design separately for its block and sub-block classifications. For each classification the A-efficiency is

$$E = A^{\text{opt}}/A,$$

the ratio of the A-value of the A-optimal completely symmetric reference design to the A-value of the design under study. A value of 1 means the design is A-optimal for that classification. For more details see Vinayaka et al. (2026).

Usage

```
npbbb_efficiency(block_design, subblock_design, v1, v2)
```

Arguments

`block_design` A matrix whose rows are the blocks (each block being the concatenation of its sub-blocks) and whose entries are treatment labels 1, . . . , v1 (test) and v1 + 1, . . . , v1 + v2 (control).

`subblock_design` A matrix whose rows are the sub-blocks, with the same labelling convention.

`v1` Number of test treatments.

`v2` Number of control treatments.

Value

An object of class "npbbb_efficiency": a list with the following components:

- **E1**: block-classification A-efficiency.
- **E2**: sub-block-classification A-efficiency.
- **A1, A2**: A-values of the design under study (block and sub-block classifications, respectively).
- **A1_opt, A2_opt**: A-values of the corresponding A-optimal completely symmetric reference designs.
- **v1, v2**: numbers of test and control treatments.

References

Hedayat AS, Majumdar D (1984) A-optimal incomplete block designs for test treatment-control comparisons. *Technometrics*, 26, 363–370.

Stufken J (1988) On bounds for the efficiency of block designs for comparing test treatments with a control. *Journal of Statistical Planning and Inference*, 19, 361–372.

Vinayaka, Parsad R, Mandal BN, LN Vinaykumar (2026) Nested partially balanced bipartite block designs for comparing test treatments with multiple controls. *Journal of Statistical Theory and Practice*. (In press).

Examples

```
d <- construct_method4(m = 3, n = 2, v2 = 2)
npbbb_efficiency(d$block_design, d$subblock_design, v1 = d$v1, v2 = d$v2)
```

n_units

Total Number of Experimental Units of an NPBBB Design

Description

Returns the total number of experimental units of a nested partially balanced bipartite block design, $N = b_2k_2 = b_1k_1$.

Usage

```
n_units(x)
```

Arguments

x An object of class "npbbb", as returned by the construct_method* functions.

Value

A single numeric value, the total number of experimental units $N = b_2k_2 = b_1k_1$.

Examples

```
d <- construct_method4(m = 3, n = 2, v2 = 2)
n_units(d)
```

print.npbbb

Print Method for NPBBB Designs

Description

Prints a nested partially balanced bipartite block design: its construction method, design parameters, block and sub-block A-efficiencies and, optionally, the full block layout with controls displayed as $\theta_1, \theta_2, \dots$

Usage

```
## S3 method for class 'npbbb'
print(x, digits = 4, show_layout = TRUE, ...)
```

Arguments

<code>x</code>	An object of class "npbbs", as returned by the <code>construct_method*</code> functions.
<code>digits</code>	Number of significant digits used when printing the A-efficiencies.
<code>show_layout</code>	Logical; if TRUE the block and sub-block layouts are printed.
<code>...</code>	Further arguments passed to or from other methods.

Value

The object `x`, invisibly.

Examples

```
d <- construct_method4(m = 3, n = 2, v2 = 2)
print(d)
print(d, show_layout = FALSE)
```

```
print.npbbs_efficiency
```

Print Method for NPBBB Efficiency Objects

Description

Prints the block and sub-block A-efficiencies of a nested partially balanced bipartite block design held in an object of class "npbbs_efficiency".

Usage

```
## S3 method for class 'npbbs_efficiency'
print(x, digits = 4, ...)
```

Arguments

<code>x</code>	An object of class "npbbs_efficiency", as returned by <code>npbbs_efficiency</code> .
<code>digits</code>	Number of significant digits used when printing the A-values and A-efficiencies.
<code>...</code>	Further arguments passed to or from other methods.

Value

The object `x`, invisibly.

Examples

```
d <- construct_method4(m = 3, n = 2, v2 = 2)
print(d$efficiency)
```

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